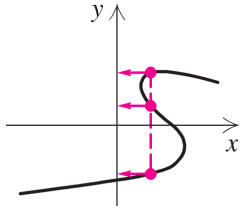


The Graph of a Function

Not every collection of points in the xy -plane represents a function. Remember, for a function, each number x in the domain has exactly one image y in the range. The graph of the function must satisfy the *vertical line test*.

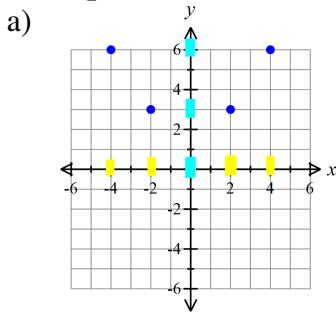
Vertical Line Test



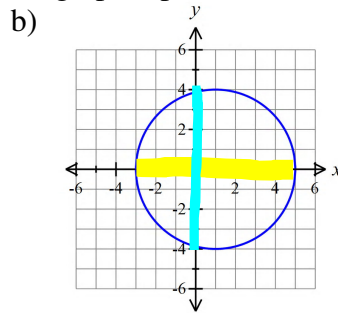
If it is possible for a vertical line to cross a graph more than once, then the graph is not the graph of a function.

The graph at left is not a function because three y -values correspond to one x -value.

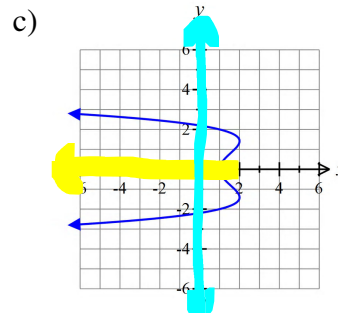
Examples: Decide whether each graph represents a function. Then find the domain and range.



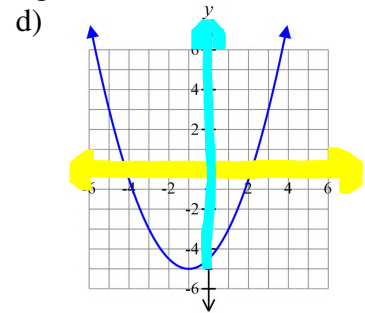
Function
Domain: $\{-4, -2, 0, 2, 4\}$
Range: $\{0, 3, 6\}$



Not a function
Domain: $[-5, 5]$
or $\{x \mid -5 \leq x \leq 5\}$
Range: $[-4, 4]$
or $\{y \mid -4 \leq y \leq 4\}$

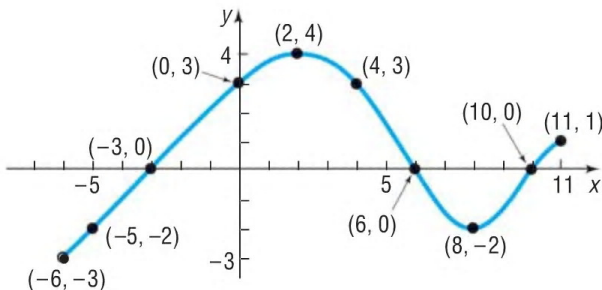


Not a function
Domain: $(-\infty, 2]$
or $\{x \mid x \leq 2\}$
Range: $(-\infty, \infty)$ or \mathbb{R}



Function
Domain: $(-\infty, \infty)$ or \mathbb{R}
Range: $[-5, \infty)$
or $\{y \mid y \geq -5\}$

Example (#9 on homework):



- a) Find $f(0)$ and $f(-6)$.
 $f(0) = 3$
 $f(-6) = -3$
what is y when x=0?
- b) Find $f(6)$ and $f(11)$.
 $f(6) = 0$
 $f(11) = 1$
- c) Is $f(3)$ positive or negative? *positive*
(graph is above x-axis at x=3)
- d) Is $f(-4)$ positive or negative? *negative*
(graph is below x-axis at x=-4)
- e) For what values of x is $f(x) = 0$? $x = -3, 6, 10$
what is x when y=0?

- f) For what values of x is $f(x) > 0$? $(-3, 6) \cup (10, 11]$
where is graph above x-axis?
or $\{x \mid -3 < x < 6 \text{ or } 10 < x \leq 11\}$
- g) What is the domain of f ? $[-6, 11]$ or $\{x \mid -6 \leq x \leq 11\}$
- h) What is the range of f ? $[-3, 4]$ or $\{y \mid -3 \leq y \leq 4\}$
- i) What are the x -intercepts? $-3, 6, 10$
- j) What is the y -intercept? 3
- k) How often does the line $y = 1/2$ intersect the graph?
horizontal line *3 times*
- l) How often does the line $x = 5$ intersect the graph?
vertical line *once*
- m) For what values of x does $f(x) = 3$?
what is x when y=3? $x = 0 \text{ and } 4$
- n) For what values of x does $f(x) = -2$?
what is x when y=-2? $x = -5 \text{ and } 8$

Example: $f(x) = \frac{x^2 + 2}{x + 4}$

a) Is the point $(1, 3/5)$ on the graph of f ?

$$\frac{3}{5} \stackrel{?}{=} \frac{1^2 + 2}{1 + 4} \quad \frac{3}{5} = \frac{3}{5} \quad \checkmark \quad \boxed{\text{yes}}$$

b) If $x = 0$, what is $f(x)$? What point is on the graph of f ?

$$f(0) = \frac{0^2 + 2}{0 + 4} = \frac{2}{4} = \frac{1}{2} \quad \text{Point on graph: } \boxed{(0, 1/2)}$$

c) If $f(x) = 1/2$, what is x ? What point(s) are on the graph of f ?

$$\frac{1}{2} = \frac{x^2 + 2}{x + 4} \quad \text{cross multiply} \quad x + 4 = 2(x^2 + 2) \quad 2x^2 - x = 0 \quad x(2x - 1) = 0 \quad \boxed{x = 0 \text{ or } x = 1/2} \quad \boxed{(0, 1/2) \text{ and } (1/2, 1/2)}$$

d) What is the domain of f ?

$$x + 4 \neq 0 \quad \boxed{\{x \mid x \neq -4\} \text{ or } (-\infty, -4) \cup (-4, \infty)}$$

e) List the x -intercepts, if any, of the graph of f .

$$\begin{aligned} y &= 0 & 0 &= \frac{x^2 + 2}{x + 4} & x^2 + 2 &= 0 & x^2 &= -2 & x &= \pm \sqrt{-2} \end{aligned} \quad \begin{array}{l} \text{imaginary} \\ \text{no } x\text{-intercepts} \end{array}$$

f) List the y -intercept, if there is one, of the graph of f .

$$\begin{aligned} x &= 0 & \text{see part b).} \\ & \boxed{1/2} \end{aligned}$$

Example: A golf ball is hit with an initial velocity of 130 feet per second at an inclination of 45° to the horizontal. In physics, it is established that the height h of the golf ball is given by the function

$$h(x) = \frac{-32x^2}{130^2} + x \quad \text{where } x \text{ is the horizontal distance that the golf ball has traveled.}$$

a) Determine the height of the golf ball after it has traveled 100 feet, 300 feet, and 500 feet.

$$h(100) = \frac{-32(100)^2}{130^2} + 100$$

$$\boxed{h(100) = 81.1 \text{ ft}}$$

$$h(300) = \frac{-32(300)^2}{130^2} + 300$$

$$\boxed{h(300) = 129.6 \text{ ft}}$$

$$h(500) = \frac{-32(500)^2}{130^2} + 500$$

$$\boxed{h(500) = 26.6 \text{ ft}}$$

b) How far was the golf ball hit?

When it hits the ground, its height is 0.

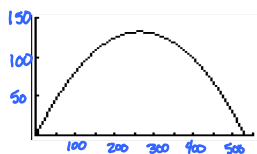
$$16,900(0) = \left(\frac{-32x^2}{16,900} + x \right) 16,900$$

$$0 = x(-32x + 16,900)$$

$$x = 0 \text{ or } x = \frac{16,900}{32} = \boxed{528.125 \text{ ft}}$$

$$0 = -32x^2 + 16,900x$$

c) Using a graphing calculator, graph the function $h = h(x)$.



d) How far has the ball traveled when it reaches its maximum height? What is its maximum height?

The ball travels 264.1 ft forward to reach its maximum height of 132.0 ft

